ME 125PL Final Project

Model of Tidal Turbine

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Santa Barbara

June 7, 2020
ME 125PL

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Contents

1 Introduction

Unfortunately in this course, we did not discuss tidal energy extraction nearly as much as I thought we were going to. So for my final project, I wanted to model a tidal turbine instead of a wind turbine. I followed the design and performance characterization procedure as laid out in the assignment for a HAWT, except I modeled the turbine in water and added a few extra components to my project. In the process of researching for this project, I realized that no one has done a full analysis or plan for a tidal farm in Santa Barbara (or all of California for that matter). This made the analysis a bit more difficult, but also opens the door for continuing work on this project in the future.

2 Proposed Location, Flow Speed and Basic Design

Since I am completing this project as a part of a UC Santa Barbara course, I thought I would model a potential tidal farm to sit in the Santa Barbara Channel (in between Santa Barbara and the Channel Islands).

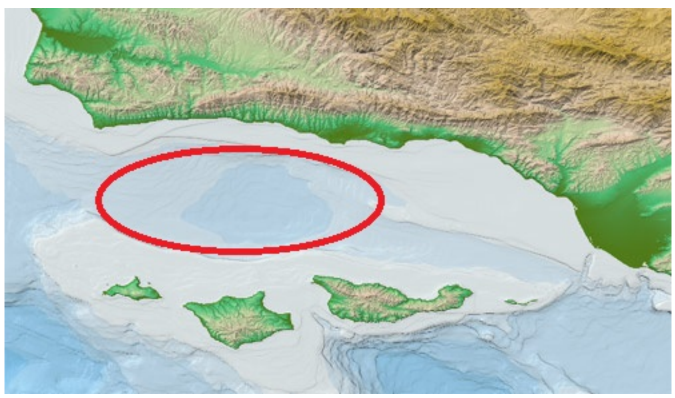

Figure 1: Santa Barbara Channel

The flow speed of water caused by the tides is determined by the depth of the ocean in that area and the height of the tide difference.

$$U = h\sqrt{\frac{g}{d}}$$

The depth of the Santa Barbara Channel closer to shore is about 60m. The average height difference between the tide extremes was a bit more difficult to find, as the tide heights at the beach are not the same as the tide differences off of the coast, and I am not very skilled at reading nautical charts. An educated guess of an average height difference of 2m was reached. This gives a flow rate of: $U = 0.8$ m/s, which is within the range of tidal stream flows harnessed by current tidal turbines.

The size of tidal turbines can be much smaller than wind turbines and produce the same amount of power because water is so much more dense than air. For this reason, I chose the turbine to have a diameter equal to 10m and will later on compare it to a wind turbine which produces the same amount of power but with a much larger size. In the project assignment, we were instructed to use a constant tip speed ratio of $\lambda = 6$ as well. After researching current tidal turbines being tested today, I noticed many of them used two blades. I chose for my model to have two blades as well.

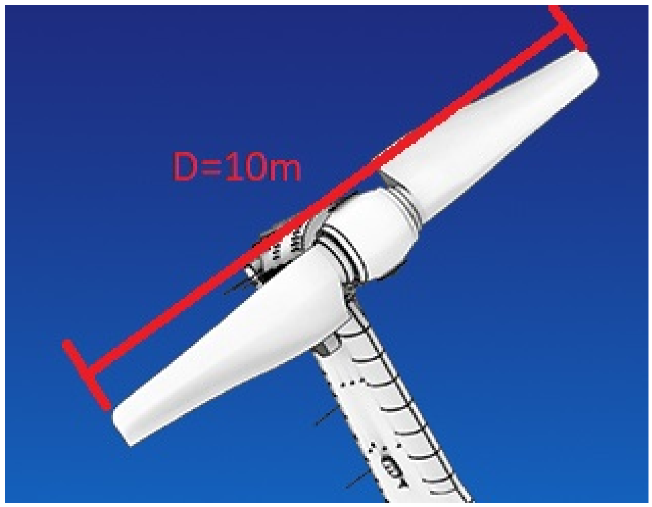

Figure 2: Basic Turbine Design

3 Airfoil Selection

There is limited research on the best airfoils to use for tidal turbines. The few papers I found resorted to designing their own airfoils, none of which had reliable data to calculate performance characteristics of an actual turbine. Thus, I chose an airfoil from the S8 family which is normally used for HAWT. Because water is much denser than air, I tried to select a thicker airfoil for structural purposes. I decided upon the S809-nR airfoil because it preformed well at high Reynolds numbers. It also had a unique shape which I thought looked cool. The outline of the airfoil along with its performance characteristics are shown below:

3.1 Finding Reynolds Number at $\frac{r}{R} = \frac{2}{3}$ and Airfoil Data

The Reynolds number at the tip of the blade is found using:

$$Re_{tip} = \frac{\rho \omega R c}{\mu}$$

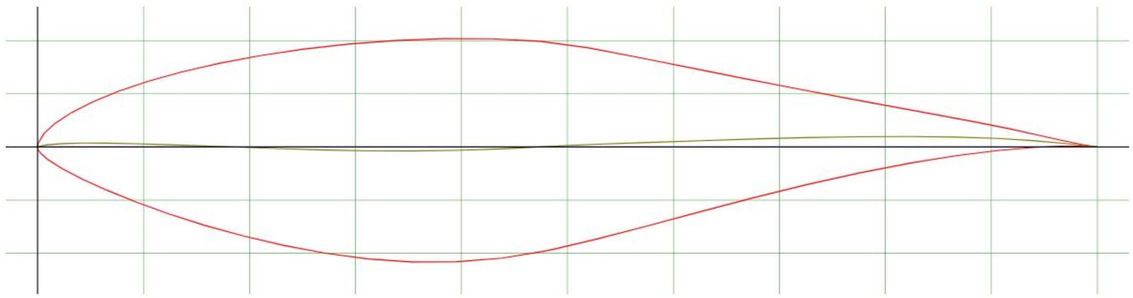

Figure 3: S809-nR Airfoil

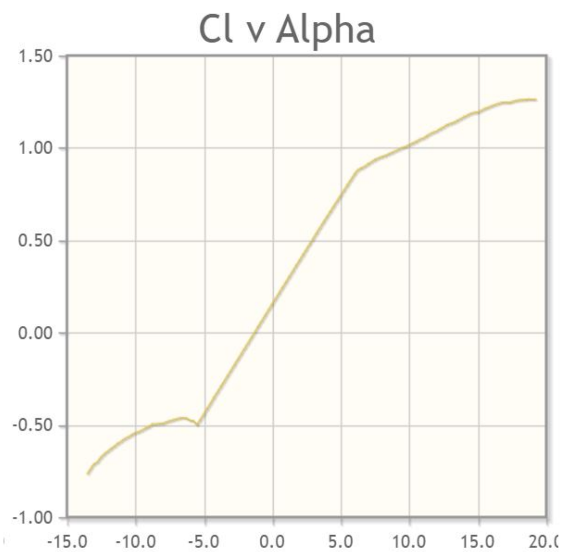

Figure 4: C_L vs α

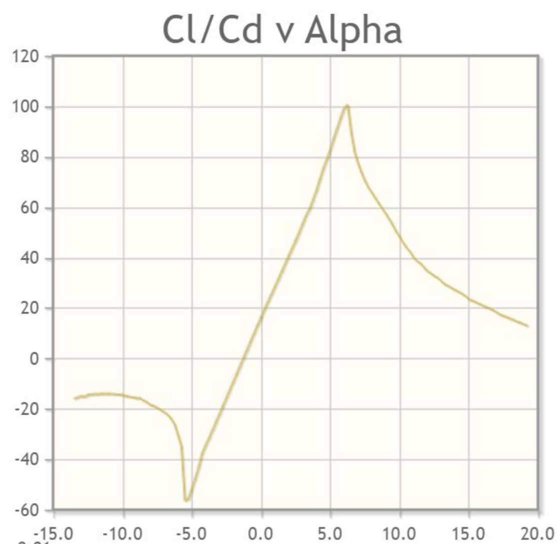

Figure 5: $\frac{C_L}{C_d}$ vs α

Where R equals the radius of the turbine and c represents the chord length. This can be rearranged to solve for the Reynolds number anywhere along the blade.

$$Re = \frac{\rho x \lambda V_0 c}{\mu}$$

Where $x = r/R$ is the nondimensional distance along the blade. By plugging in the values for $x = 2/3$, we obtain $Re \approx 2,000,000$. The performance characteristics of the airfoil were then found at this Reynolds number using *airfoiltools.com*. The operating point was chosen to be when the maximum $\frac{C_L}{C_d}$ was reached, which corresponded to $\alpha = 6°$. The following characteristics were found at this angle of attack:

$$c_L = 0.9$$

$$\frac{c_L}{c_d} = 90$$

4 Rotor Shape

Once the airfoil design is chosen, the shape of a rotor is determined by its chord length and pitch angle. These are determined from the following equations:

$$c(x) = \frac{16R\pi}{9c_L B\lambda^2 x}$$

$$\theta(x) = \frac{2}{3\lambda x} - \alpha$$

As stated in the assignment, I took $a = \frac{1}{3}$ and took tip effects into account. Tip effects are caused by vortices which shed off of the ends of the airfoil and cause turbulence and decreased performance. This effect can be characterized by a factor F and reduces the effective amount of blades on the turbine. I defined an effective B such that $B_{eff} = B/F$ where F equals:

$$F = \frac{2}{\pi}cos^{-1}(exp(\frac{-(1-x)B\lambda}{2(1-a)}))$$

The shape of rotor was found by these equations for $0.25 < x < 1$, as the equations break down as x gets too small. This region was manually adjusted to have constant values to simplify the shape and give more structural strength. The graphs of tip effect ($F(x)$), chord length ($c(x)$), and pitch angle ($\theta(x)$) are shown below:

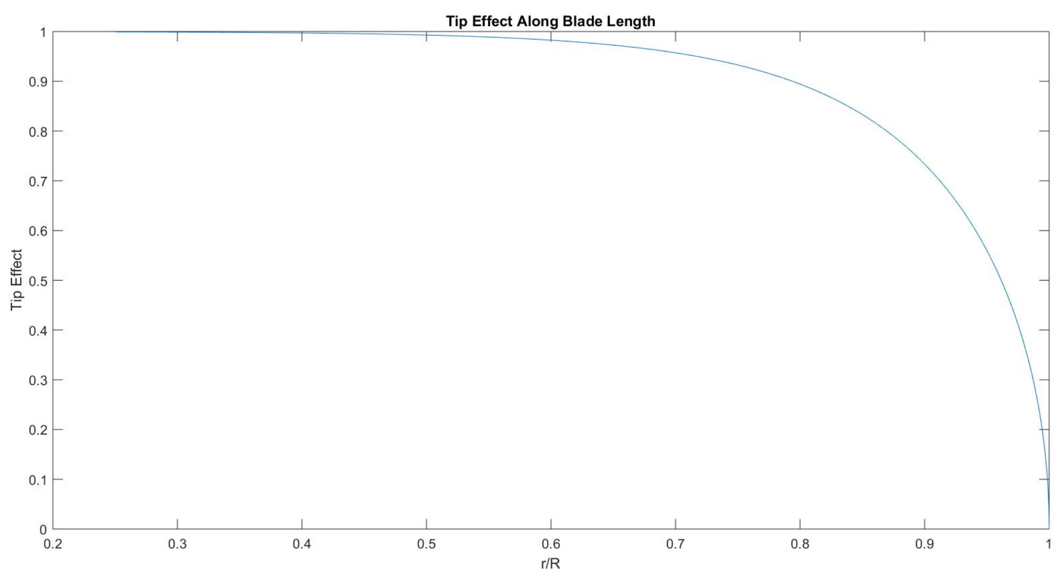

Figure 6: Tip Effect as Function of r/R

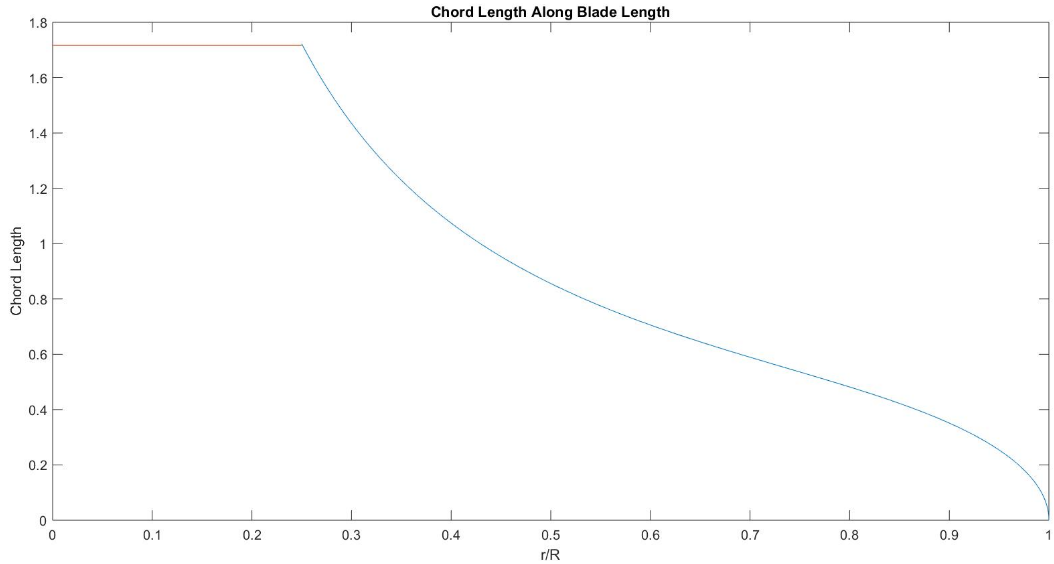

Figure 7: Chord Length in meters as Function of r/R

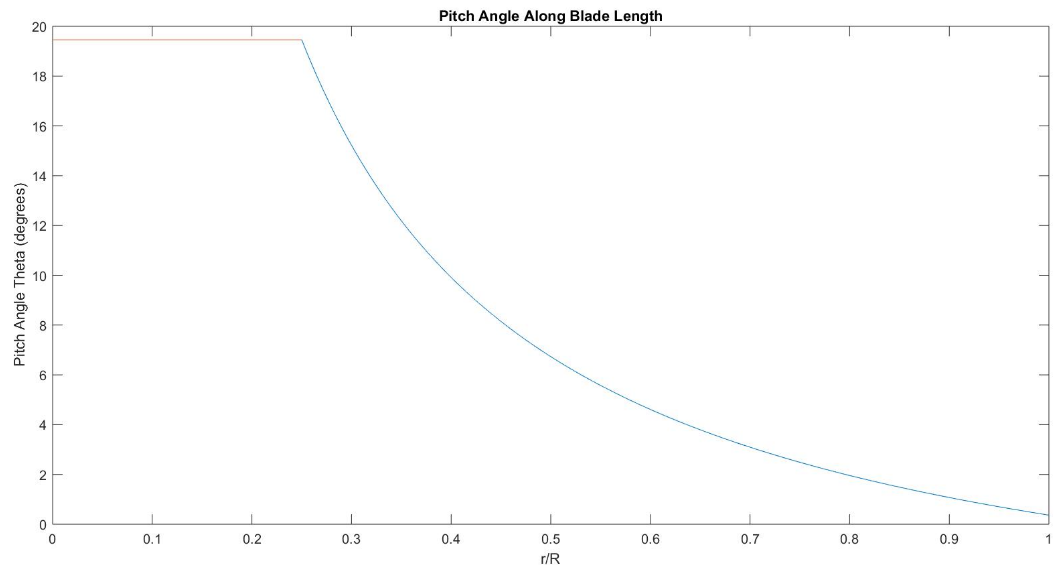

Figure 8: Pitch Angle (θ) as Function of r/R

5 Power, Thrust and Torque Calculations

To calculate the total power, thrust and torque as well as their corresponding coefficients, the following formulas were used. For simplicity, a was assumed to be equal to 1/3.

$$C_p = \int_0^1 \left(\frac{16F}{9\lambda}\right)\left((1-a)^2 + (\lambda x(1-a'))^2\right)\left(sin(\phi) - \frac{cos(\phi)}{c_L/c_d}\right)dx$$

$$a' \approx \frac{a(1-a)}{\lambda^2 x^2}$$

$$C_T = \int_0^1 8a(1-a)Fxdx$$

$$C_q = \frac{c_p}{\lambda}$$

$$P = C_p \frac{1}{2} \rho V_0^3 A$$

$$T = C_T \frac{1}{2} \rho V_0^2 A$$

$$Q = C_q \frac{\pi}{2} \rho V_0^2 R^3$$

The following performance characteristics were found using MATLAB integration. The integral for C_p was found to blow up to infinity when $x < 0.1$, so the lower limit of that integral was changed to 0.1. These numbers make physical sense as they are all less than the upper limits on performance as given by actuator disc theory.

$$C_p = 0.5298 \qquad \text{Power} = 1.0652\text{x}10^4 W$$

$$C_T = 0.7663 \qquad \text{Thrust} = 1.9258\text{x}10^4 N$$

$$C_q = 0.0883 \qquad \text{Torque} = 1.1096\text{x}10^4 N/m$$

5.1 Comparing Power to Offshore Wind Turbine

The average offshore wind speed in the Santa Barbara Channel is about 5 m/s. In order for an offshore wind turbine (with the same power coefficient) to generate the same amount of power as this tidal turbine, the wind turbine's diameter would have to be about twice as large. This is assuming that the offshore wind turbine is producing energy just as often as the tidal turbine, which is unlikely because tidal flow is much more predictable and reliable than the wind. The tidal heights used in all of these calculations is an underestimate as well, so the tidal flow is probably significantly faster than expected and can generate more energy.

6 Water Flows Exceeding Rated Speed $U = 0.8m/s$

When the flow rate increases and exceeds the rated speed, the power generated and the angular velocity of the rotor need to be kept constant. This is done by rotating the blades by an angle "Ψ" so that less lift is generated and the the turbine stays spinning at the same rate. This angle was found by working backwards from the power generated to obtain a new power coefficient for that water flow speed, and then numerically solving for ϕ and thus Ψ by the relation: $\Phi = (\Psi + \theta) + \alpha$. The angle of attack α and the extra rotation angle Ψ remain constant along the length of the blade whereas the rest of the angles change according to the design shape.

Since the rotor is spinning at the same rate for a greater incident flow speed, the tip speed ratio λ will change by the relationship $\lambda = 4.8/V_0$. The following graphs for Ψ and thrust as functions of flow velocity are shown below:

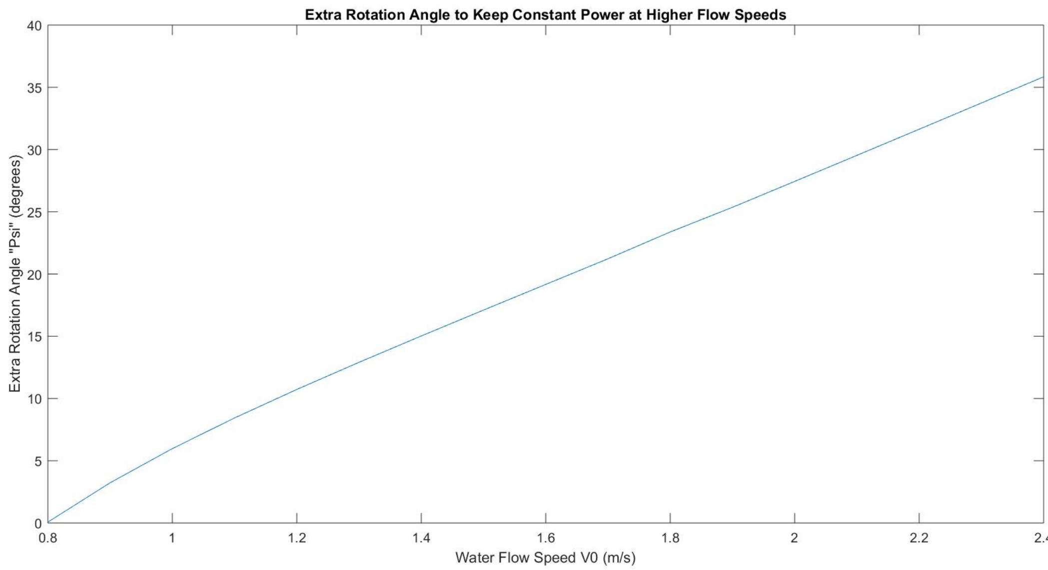

Figure 9: Ψ as Function of $V_0 > 0.8m/s$

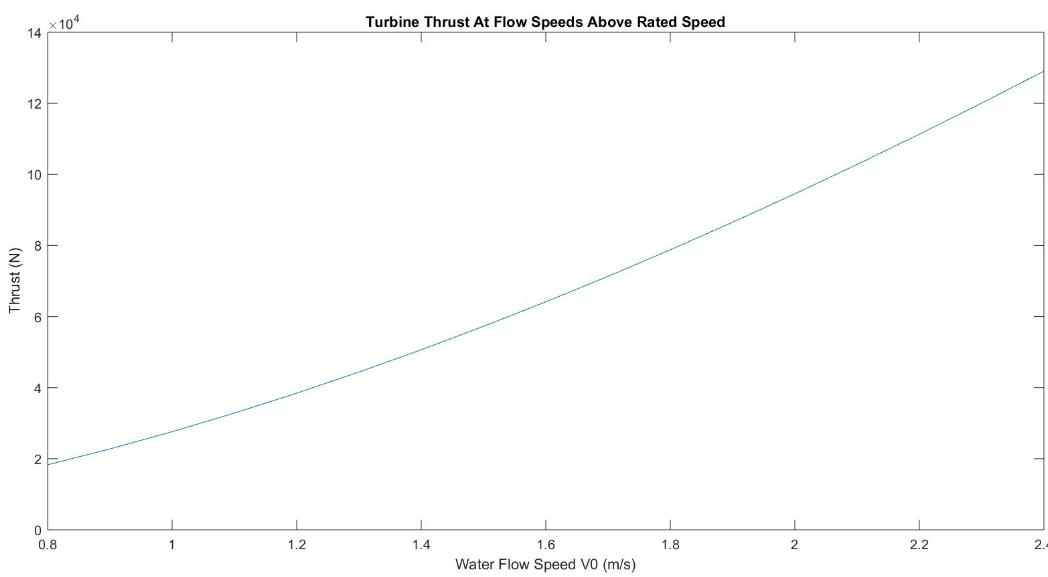

Figure 10: Thrust as Function of $V_0 > 0.8m/s$

7 Conclusion

This entire project was completed under the assumption that tidal turbines can be treated almost exactly like wind turbines. It was fascinating to see how much energy a tidal turbine can generate at such small flow speeds, and can afford to be much smaller than their wind counterparts. For these reasons, tidal energy is a potentially viable and reliable source of energy for the Santa Barbara. This source of energy would help supplement wind and solar energy, both of which are intermittent and unpredictable.

If I were to continue work on this project, I would want to model the wakes created by these turbines and find the optimal layout for a tidal farm in the area. I would also like to look more into the tidal stream data for the Santa Barbara Channel area as the data I used wasn't much more than an educated guess. There was a lot of uncertainty in the choice of airfoil for this project as well, so model testing and comparison of prototype turbines using different airfoils in a water tunnel would be a great next step too.