Shallow Water Waves Striking a Pier

COMSOL Multiphysics Final Project

Produced By

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Contents

1 Abstract

The goal of this project was to explore the different variables that effect the force that waves can exert on structures, and ultimately apply these forces to a modeled wooden pier and examine its response. This simulation utilizes the Shallow Water Equations module in COMSOL to generate waves and cause them strike a series of columns. The force a wave exerts was seen to increase quadratically with the initial height of the wave, and an optimal wave propagation distance was observed for which a wave exerts it's peak force. A 1m tall wave was modeled to strike three pier legs, and the measured stress was less than the ultimate tensile strength of the material. An eigenfrequency analysis was also preformed on the pier structure, which determined that a resonance scenario caused by sinusoidal wave forces was unlikely.

2 Introduction

Ocean waves can exert large forces on objects that are immersed and anchored to the sea floor, which can tear apart structures if they are not designed to withstand this type of environment. The issue is that surface ocean waves are difficult to analyze using hand calculations as they are complex phenomenon. COMSOL's Shallow Water Equations module is a great tool to investigate ocean waves, as a simulated wave tank can be created easily and the input parameters readily varied. In this report, a wave tank was simulated which represented a flat, nearshore ocean environment. Several columns were then placed in the wave tank and the forces exerted on these columns measured as several different variables were altered. These forces were then applied to a six-legged structure with a flat platform on top which represents a pier. The response of this pier structure was analyzed to see if the pier was fit to withstand waves of a certain height.

Ocean waves have a regular periodicity, meaning they can exert forces in a cyclic manner. For anyone who has studied vibrations before, this situation screams resonance. If a structure whose resonant frequency was in the range of possible forceful ocean waves was placed into a shallow ocean water environment, the ocean could continuously dump energy into the pier until it tore itself apart. For this reason, an eigenfrequency analysis was performed on the pier structure and the frequency of the displacement of the pier was measured. The stresses experienced by the pier were also measured to see how they compared to the ultimate tensile strength of the material used (American red oak specifically, as most piers are made of wood).

The overall purpose of this simulation was to gain insight on the forces and stresses associated with ocean waves. I am genuinely interested in ocean phenomenon as my capstone project will constitute of components placed in the ocean for extending periods of time. Although this simulation does not directly model my capstone project, it still provides an intuition about the forces and movements structures placed in the ocean will experience.

3 Procedure

3.1 Governing Equations

3.1.1 Shallow Water Equations (SWE)

The shallow water equations (equations 1 and 2) are derived from the Navier-Stokes equation for situations where the horizontal length scale (direction of wave propagation) is much greater than the vertical length scale (depth of fluid). The velocity of the fluid in the vertical direction is assumed to be negligible compared to the horizontal velocity when solving the shallow water equations. The vertical velocity can be solved for in a following step using the continuity equation (3).

$$\frac{\delta h}{\delta t} + \frac{\delta}{\delta x}((H+h)u) + \frac{\delta}{\delta y}((H+h)v) = 0 \tag{1}$$

$$\frac{\delta(hu)}{\delta t} + \nabla \cdot (hu \times u + \frac{1}{2}gh^2 l) = -gh\nabla h_b + F \tag{2}$$

$$\nabla \cdot u = 0 \tag{3}$$

Equation (1) describes mass continuity, equation (2) describes momentum conservation and equation (3) describes continuity of an incompressible fluid. Much of the right side of equation (2) is ignored in this simulation as the bottom contour (h_b) is constant and there are no other body forces (F) acting on the fluid such as a coriolis force.

3.1.2 Solid Mechanics

Since the force on the legs of the pier changes over time and the legs are struck by the wave at different times, time dependent solid mechanics was used in this simulation.

$$\rho\frac{\delta^2 u}{\delta t^2} = \nabla \cdot S + F_v \tag{4}$$

In equation (4), S represents the stress matrix and F_v is the volume force matrix. This equation is solved to calculate the stress on each element throughout the simulated time.

3.1.3 Eigenfrequency Analysis

The vibrations and responses of a structural system with multiple degrees of freedom can be described using the following equation:

$$\mathbf{M}\ddot{u} + \mathbf{C}\dot{u} + \mathbf{K}u = \mathbf{F}(t) \tag{5}$$

Where $\mathbf{M}$ represents the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix and $\mathbf{F}(t)$ is the forces matrix.

3.2 COMSOL Implementation

3.2.1 Wave Tank Shallow Water Equations

Two separate model wave tanks were created for this project. The first tank contained only one column (figure 1) and a bulk fluid depth of $0.3m$, as the effects of wave height and wave propagation distance were analyzed first. The wave tank was then modified to include 3 columns (figure 2) and the water depth changed to $3m$ to simulate a pier sitting in a body of water. The cross-sectional shape of the columns was also changed from square to circle for this simulation as this more accurately reflects the shape of most pier legs.

In both tanks, the columns are modeled as open spaces in the tank with walls such that the water medium cannot exist in this space. Since the "Shallow Water Equations" COMSOL module can only be applied with 2D geometries, the columns and walls of the tank are modeled as having infinite height.

Initially, the tank is divided into two regions with different initial values of water height. The largest region of the tank (contains the columns) is set to have some initial water height that represents the depth of the bulk fluid. The water height of the smaller region on the bottom of figures 1 and 2 represents the wave height. It is set to have an initial value equal to the bulk fluid depth plus some wave height. After t=0, these two regions are suddenly allowed to equalize, which creates a wave traveling in the upwards direction seen in figures 1 and 2. The medium used in the "Shallow Water Equations" COMSOL module is fresh water by default, and was not changed throughout this simulation.

To vary the wave height that struck the column(s), the initial value of the water height in the bottom region was changed using a parametric sweep. To vary the distance the wave traveled before striking the column(s), the y-axis position of the column was parameterized and changed through a sweep.

To calculate the force exerted by the wave striking the column(s), an integration non-local coupling was used to integrate the force in the y-direction (Fpy.swe) along one side of the column. This calculation was done in the "Global Variables" section so that the values could be saved and applied to the pier's solid mechanics. The values of force vs time for each column were saved in a results table.

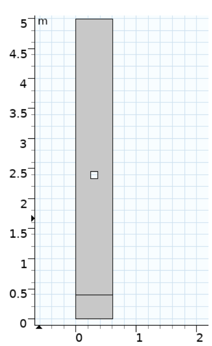

Figure 1: SWE Setup 1 Column

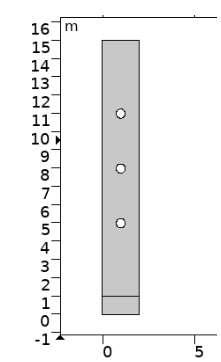

Figure 2: SWE Setup 3 Columns

3.2.2 Pier Solid Mechanics

The Solid Mechanics COMSOL module was used to model the pier. The legs of each column were fixed as seen in figure 3, which represents a pier anchored into the surrounding beach. To apply the forces exerted by the wave, an x-y work-plane was introduced which divided the domain of each pier leg in two. The height of the work plane was defined by the height of the bulk fluid depth (3m in the case of this simulation). Boundary loads were then created for the three pairs of pier legs which would exert the time dependent wave forces derived from the SWE simulation.

In order for the forcing functions to be continuous, the force vs time data saved to results tables from the SWE simulation had to be interpolated. This was done with an interpolation function in COMSOL which created a linear function between each data point in the table. These interpolated functions were then called in the boundary load nodes as time dependent forces acting in the y-direction (figure 4).

In order to calculate the displacement of the top platform of the pier, another integration non-local coupling was applied to the side edge of the platform. The internal COMSOL displacement variable in the y-direction, v, was integrated over the side edge face of the platform, and this value was divided by the area of that face.

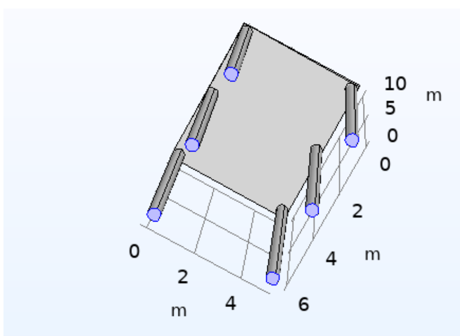

Figure 3: Fixed Constraints of Pier

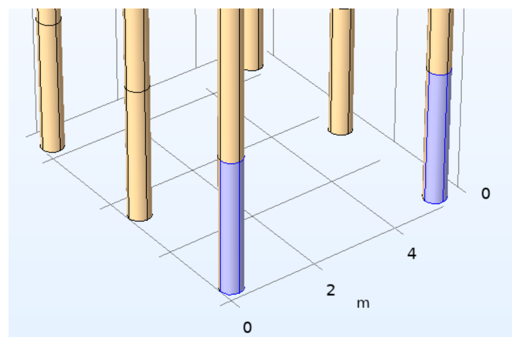

Figure 4: Boundary Load Applied to 1st Column Pair

3.2.3 Meshing and Material

For all components in this simulation, physics-controlled meshes were used with an element size of normal. These meshes included suitable numbers of elements in critical areas (around the perimeter of the columns in SWE and along length of legs of the pier). There were no discontinuities seen in the results of these simulations indicating any issues with the normal physics controlled meshing. There were no material selections made for the SWE component, as water is the default. American red oak was selected for the entirety of the pier and solid mechanics component.

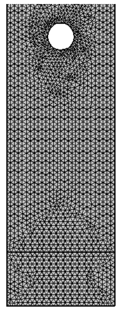

Figure 5: SWE Tank Mesh

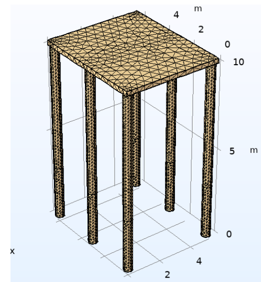

Figure 6: Pier Mesh

3.2.4 Study

The shallow water equations could only be solved using a time dependent study in COMSOL, so that study type was used for that simulation. The simulation was run from $t = 0s$ to $t = 2.5s$ with a time step of $0.01s$. A time dependent study was also used for the pier's solid mechanics with the same time step, start and end times. An eigenfrequency study step was also included for this component to find the structure's resonant frequencies. Nearly all significant ocean waves that a pier would encounter have a period between 3 and 20 seconds, so the eigenfrequency analysis was preformed about 1Hz. For convenience sake, the shallow water equations and solid mechanics were divided up into separate studies and executed one after another. This was to allow the force vs time data derived from the SWE to save to a results table and then be interpolated during the solid mechanics step.

4 Results

4.1 Effect of Wave Propagation Distance in Striking One Column

Figure 7 plots the force vs time on a column struck by waves with varying propagation distances. The column was placed at $0.5m, 1m, 2m, 3m$ and $4m$ away from the gate. As the column was moved farther away, the force on the column increased until reaching a maximum at around 1m away from the gate. After this point, the force on the column decreased as the column is moved farther from the gate. Figure 8 plots the maximum force on the column vs the propagation distance, making the relationship a bit easier to visualize.

4.2 Effect of Wave Height on Force Striking One Column

The force on the column increased as the height of water behind the gate increased as seen in figure 9. The best curve fit for this relationship was determined to be quadratic, whose equation is: $F = 243h^2 + 142h - 20.8$.

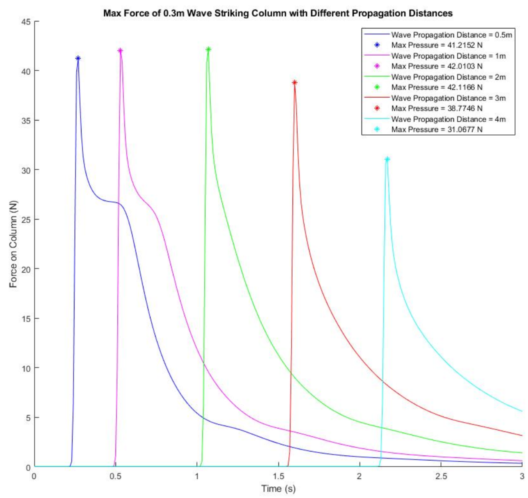

Figure 7: Force on Column vs Time for Varying Wave Propagation Distances

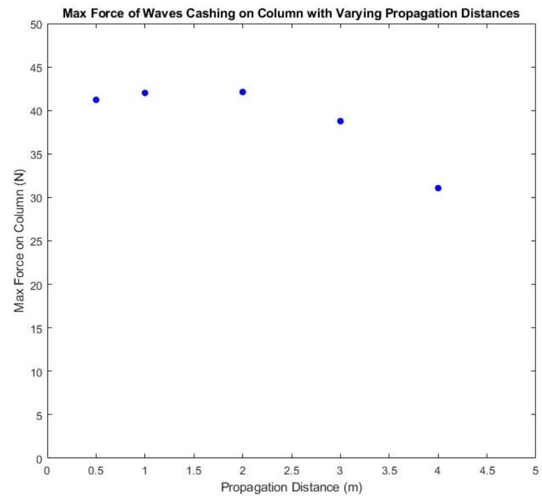

Figure 8: Max Force on Column for Varying Wave Propagation Distances

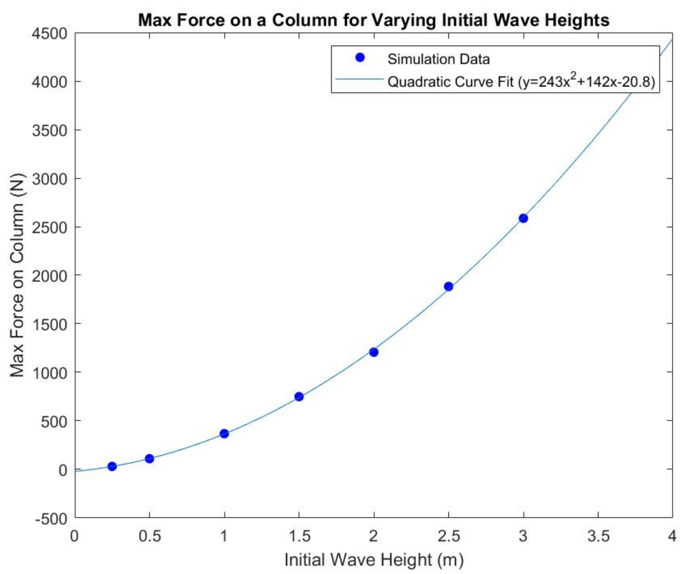

Figure 9: Initial Wave Height vs Force on One Column

4.3 Wave Striking Pier

4.3.1 Shallow Water Equations

Figure 10 shows a 1m tall initial wave traveling down the simulated water tank and striking each of the pier's columns.

The force on each row of pier columns is shown in figure 11 through time. The force on the

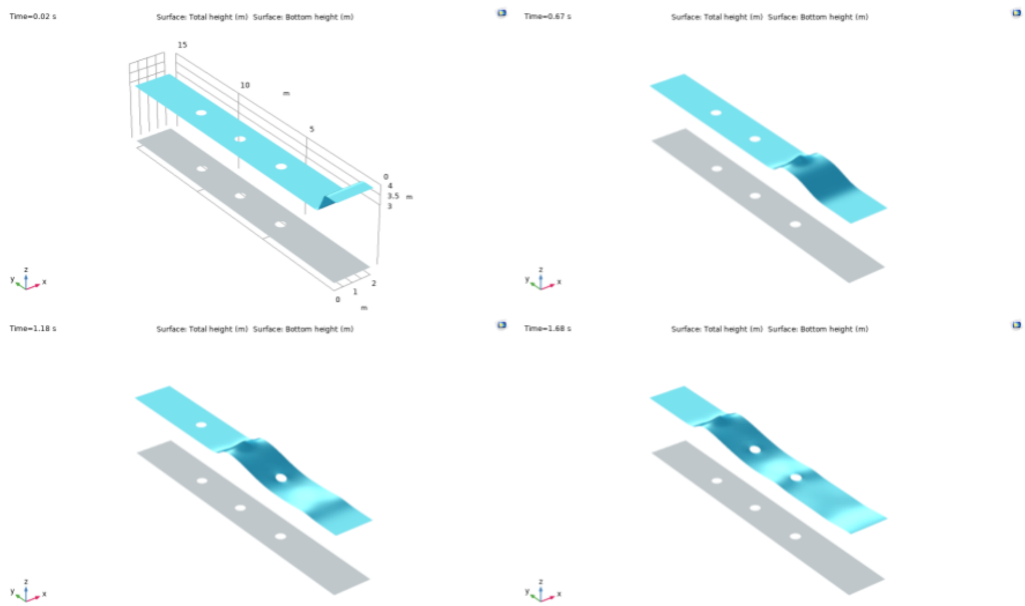

Figure 10: Water/Wave Height at Four Moments in Time

first column was the largest, and the force decreased with each subsequent column after that. Once the wave reaches each column, the shape of the force vs time peak is nearly identical for each column, although scaled down. The force exerted by the wave takes slightly longer to decrease from its peak than it does to increase to its peak. After the large spike in force, smaller force fluctuations are recorded which persist until the end of the simulation.

The force on each of the columns started off at and then returned back to $22,000N$. This $22,000N$ is a constant force applied throughout the simulation and is constant over each of the three columns.

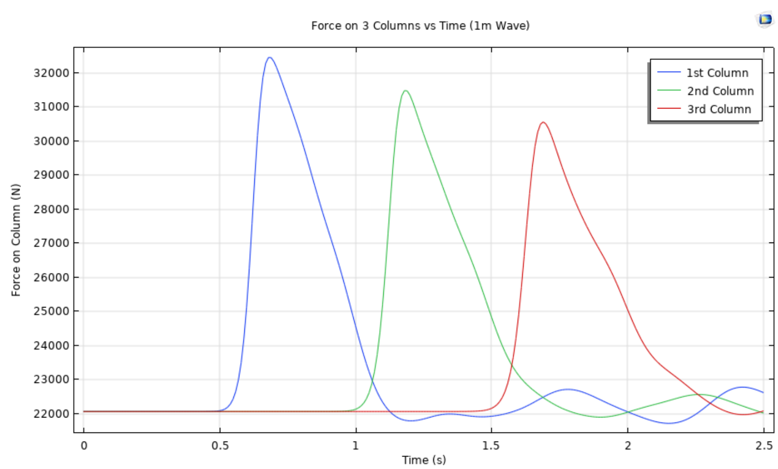

Figure 11: Force on 1st, 2nd and 3rd Columns vs Time for 1m Tall Wave

4.3.2 Time Dependent Solid Mechanics

When the time varying force derived from the shallow water study is applied to the pier for a time dependent solid mechanics study, the stress and displacement of the pier structure through time is derived. Figure 12 shows how the stress varies with time for each set of pier legs. The maximum stress in the whole simulation occurs in the first column. The max stress in the subsequent columns is slightly more than half of the max stress in the first column. These maximum stresses in the 2nd and 3rd columns are approximately equal too.

Figure 14 visualizes the stress and displacement of the pier throughout time. The wave strikes the 1st column of the pier at $t = 0.75s$, causing the entire structure to oscillate. The wave then hits the 2nd column and 3rd columns at $t = 1.15s$ and $t = 1.84s$ respectively. After each impact, the pier oscillates back and forth in the y-direction. Figure 13 shows how the top platform of the pier moves throughout time. The displacement is defined as the distance the edge of the pier travels with respect to its initial position. The displacement of the pier's platform reaches a maximum at around $1.3s$, right after the 2nd column is struck by the wave. The oscillation of the pier's platform is quite erratic and nonlinear with respect to time.

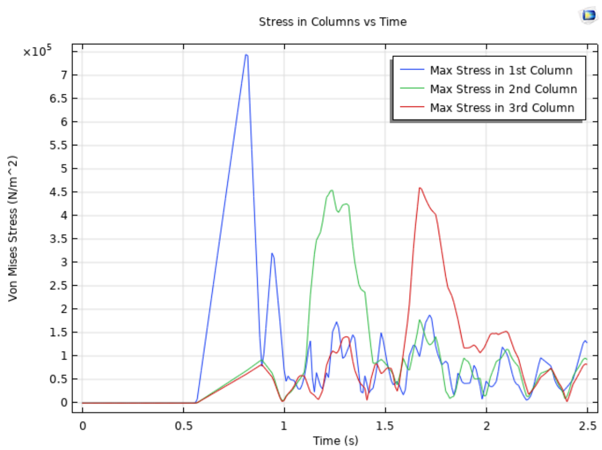

Figure 12: Max Stress in 1st, 2nd and 3rd Columns vs Time

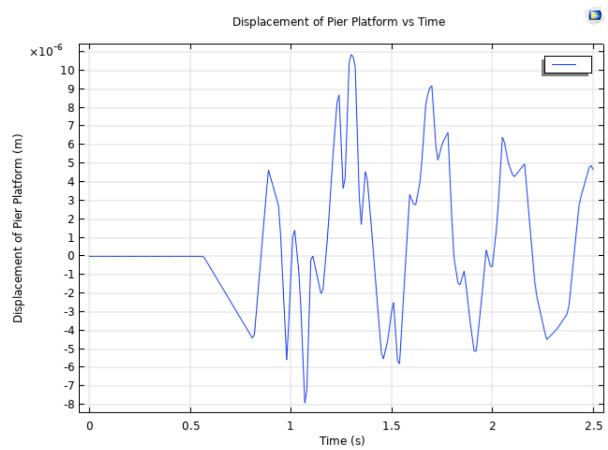

Figure 13: Pier Platform Displacement vs Time

4.3.3 Eigenfrequency Analysis

The eigenfrequency analysis returned six different modes of oscillation. However, in this situation the only mode of concern is vibrations in the y-direction, as this is the direction the time dependent forces are acting. The solution for an undamped case is seen in figure 15. The eigenfrequency for this case is $2.6189Hz$.

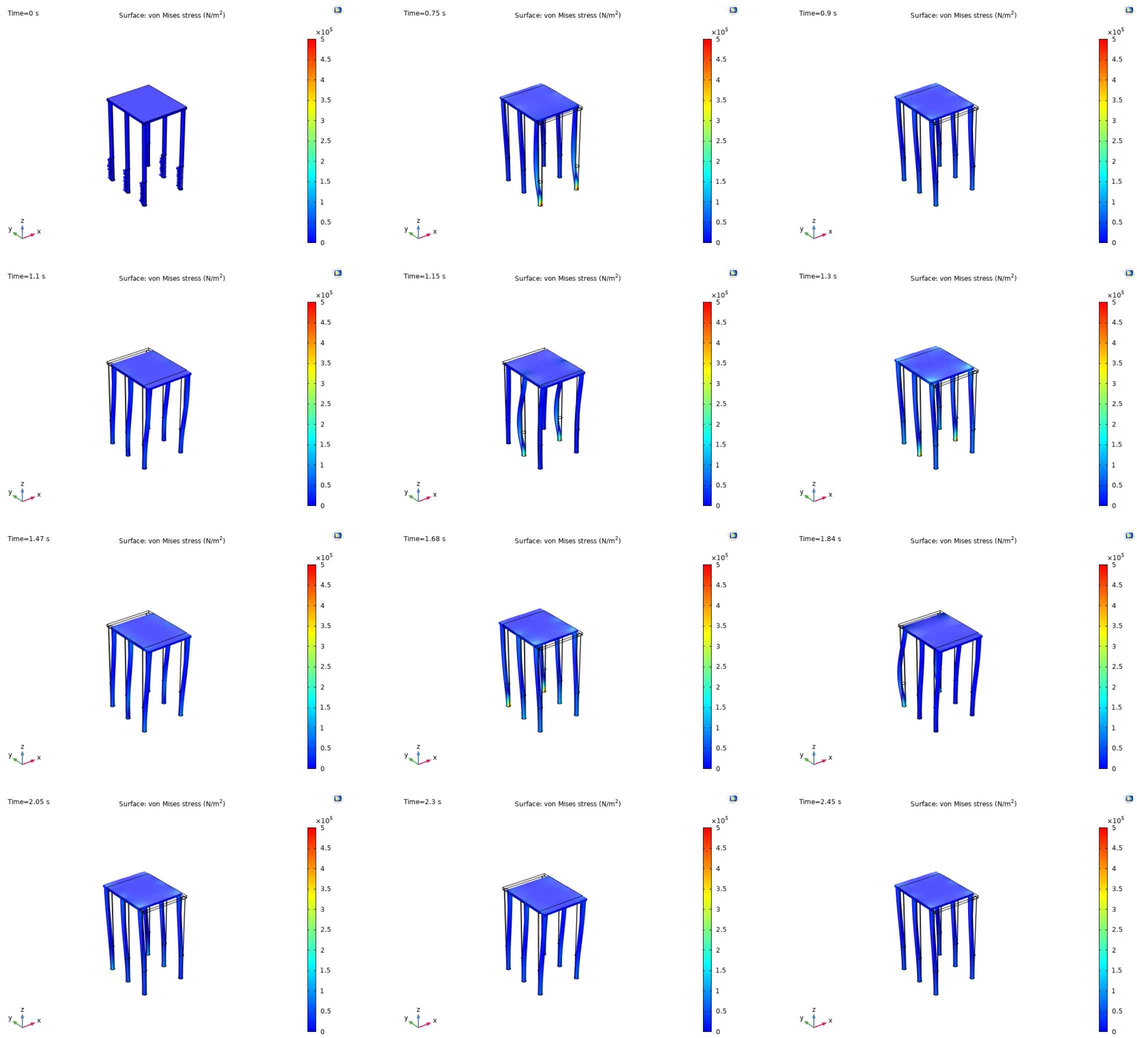

Figure 14: Stress and Displacement of Pier vs Time

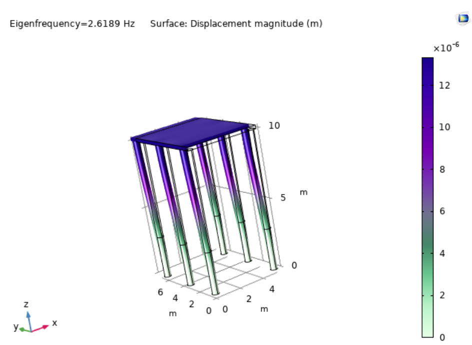

Figure 15: Eigenfrequency Analysis in Wave Direction

5 Discussion

5.1 Shallow Water Equations

The wave created in this simulation is generate by suddenly releasing a gate holding back water at a greater height. When released, there are some transient effects and some distance traveled before the wave is fully developed. When the wave is not fully developed, it does not exert as much force as it would if it were to travel farther unobstructed. After it is fully developed, the wave starts to lose energy due to disipative forces (such as interactions with the walls of the tank or turbulence) as it propagates farther. This means that there is some distance away from the gate that the wave has maximum energy and will strike the column with maximum force. This is clearly seen in figure 8 which reaches a maximum at around 1m.

As was expected, larger waves exerted larger forces on the columns in the simulation. What is most interesting is the seemingly quadratic relationship between initial wave height and force it exerts. The momentum conservation shallow water equation is a nonlinear PDE which has h^2 and h terms, and the change in momentum is proportional to the force exerted by the wave. This curve fit thus makes mathematical sense based on the governing equations.

When the simulation was run with 3 columns, the force decreased as the wave lost energy with each collision and due to other disipative forces as it traveled down the length of the tank. Smaller force fluctations were also seen after the initial peak in force, which is caused by the initial wave reflecting off the columns and walls of the tank. Smaller waves are created by these reflections which then collide with the columns. Interestingly in this simulation, there appeared to be a constant force of $22,000N$ applied equally to each column. This constant force is the hydrostatic pressure exerted by the water on the submerged column. The hydrostatic pressure is much more significant in this simulation with 3 columns as the bulk fluid depth is $3m$ instead of $0.3m$. This pressure is exerted equally in all directions, so is not meaningful in examining the dynamic forces exerted by a wave, nor important to the transient response of the pier. This constant value was subtracted out before being interpolated and applied to the solid mechanics of the pier.

5.2 Solid Mechanics and Eigenfrequency Analysis

The force vs time data derived from the Shallow Water Equations analysis varied smoothly with time as it peaked and dropped, then oscillated regularly about an equilibrium point. Once this smooth force is applied to the pier though, any semblance of a ”smooth curve” is lost when the stress on each column is analyzed. The peaks in stress for each column still occur at the same instant time as the peaks of force. Notably, the maximum stresses in the 2nd and 3rd columns are equal even though the maximum force exerted on the 2nd column is $1000N$ larger than that exerted on the 3rd column. This is most likely explained by the transient oscillations of the pier in the y-direction and the timing of the waves. After all of the waves pass the columns, the pier oscillates in a simple harmonic fashion as the time dependent forces have died off. The frequency of oscillation seen in figure 13 looks to be approximately equal to the eigenfrequency derived in section 4.3.3.

By looking closely at figure 14, one can see that when a column is struck by the wave, that pier leg deforms due bending stress first. Shortly afterwards, the pier platform responds by displacing itself in the $+y$ direction. This effect is expected, but is interestingly to visualize when analyzing a time dependent solid mechanics system over a short time step.

The maximum stress experienced by the pier was around 0.75MPa, which is less than the ultimate strength of American red oak (5.10 MPa, from Matweb). This means that the pier can survive at least one cycle of a 1m wave sequentially striking each of its legs. More analysis is needed to approximate the fatigue life of this pier, as it probably would not survive an infinite cycle of 1m tall waves striking it.

6 Conclusion

The Shallow Water Equations COMSOL module proved to be a simple tool to simulate a wave tank with controlled conditions. Under these circumstances, the waves generated were measured to peak in intensity at a non-zero optimal distance away from the wave generating point. The force was also seen to increase quadratically with the initial height of the wave. These relationships were never considered in parallel for the scope of this report. It would be intriguing to explore how the optimal propagation distance varies with initial wave height.

The forces derived from the SWE part of the simulation varied smoothly with time, whereas the solid mechanics simulation varied much more erratically due to complex oscillations. The maximum stress experienced by any part of the pier throughout the simulation was less than the yield strength of American Red Oak. However, a more in depth analysis utilizing fatigue concepts would provide insight to how the pier would hold up to real world conditions. A further study could generate many waves of different sizes reflective of a real world environment and study how the pier responds for 10^4, 10^5, 10^6 etc cycles. This stress analysis was only preformed for a 1m tall wave, so investigating the stresses involved for different initial wave heights should be considered. It would be interesting to find what initial wave height will instantly fracture the pier when it collides with one of the legs.

The eigenfrequency of the pier was calculated to be outside of the frequency range of significantly powerful ocean waves. With this pier design, resonance caused by repeated forcing from waves is extremely unlikely. Nvertheless in an additional study, it would be interesting to study what forces and timescale is required to have the pier tear itself apart due to resonance, as any forcing time can be input to COMSOL. The eigenfrequency evaluation preformed in this simulation also did not include any damping affects from the part of the pier legs that were submerged in water. This damping could affect the eigenfrequency of the structure and possibly put it in the range that ocean waves could force it at. Adding additional legs to the structure could also change the eigenfrequency of the structure and is worth investigating. It would also be useful to test whether constraining the pier's legs to the ground in a different way would affect the eigenfrequency or stresses in the structure. Specifically, the legs could be extended and the fixed constraints expanded to include some part of the vertical leg surface.